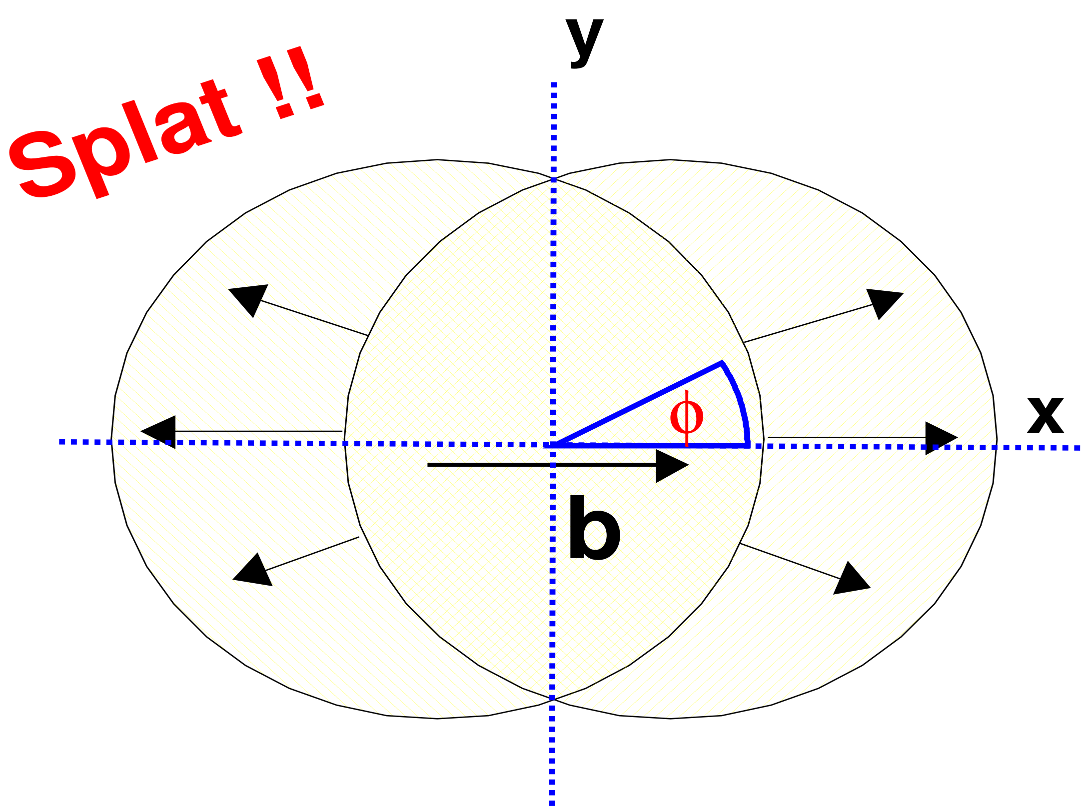
Elliptic Flow in Heavy Ion Collisions



Measure the anisotropy:

$$\frac{dN}{d\phi} = N(1 + 2\, v_2 \cos(2\phi) + \cdots)$$

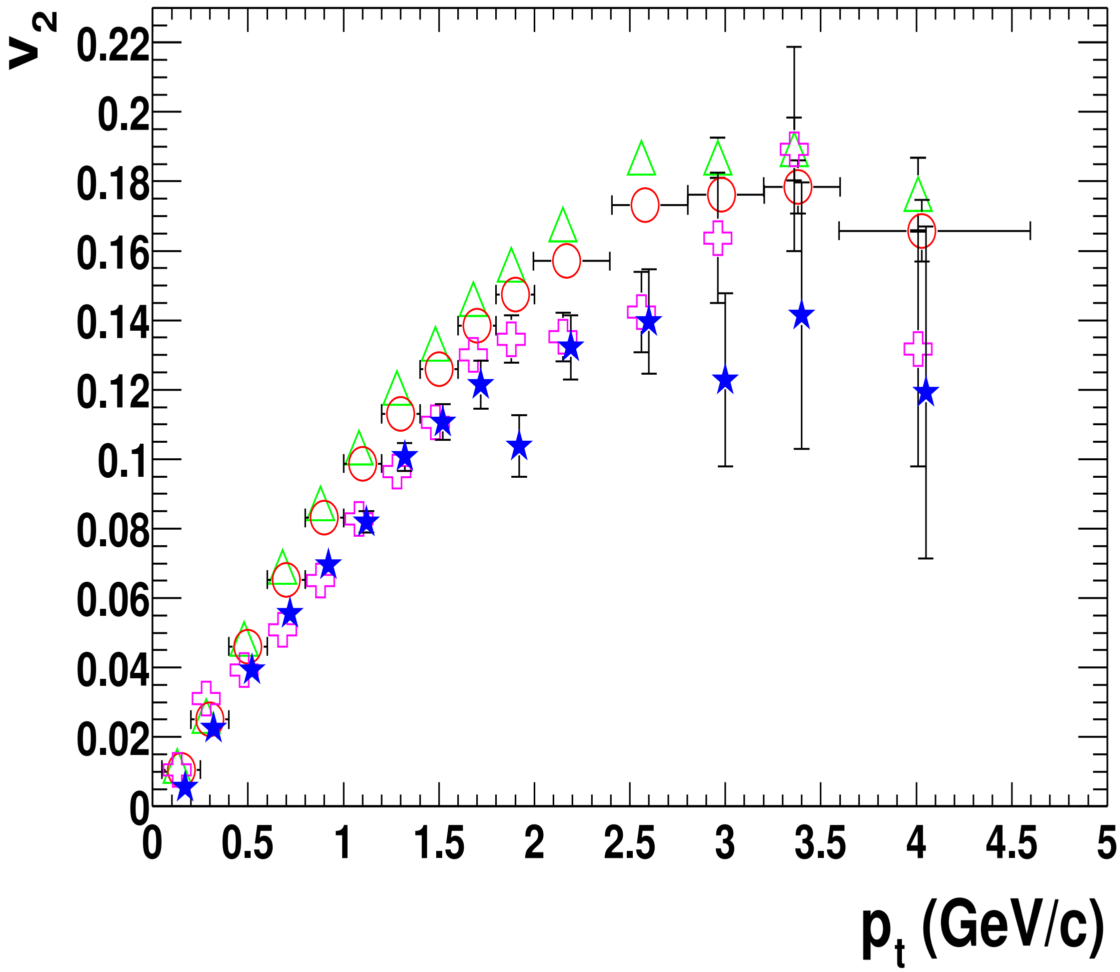
where $v_2 = \langle \cos(2\phi) \rangle$

Can also measure elliptic flow as function of transverse momentum:

$$\frac{1}{p_T} \frac{dN}{dp_T\, d\phi} \quad = \quad \frac{1}{p_T} \frac{dN}{dp_T} \; (1 + 2\, v_2(p_T) \cos(2\,\phi) \cdots)$$

Then $v_2(p_T) \equiv \langle \cos(2\,\phi) \rangle_{p_T}$.

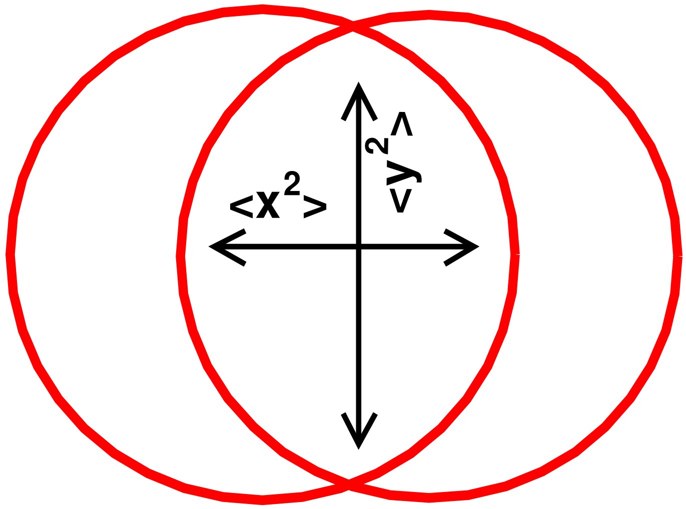
Amazing Results from RHIC:



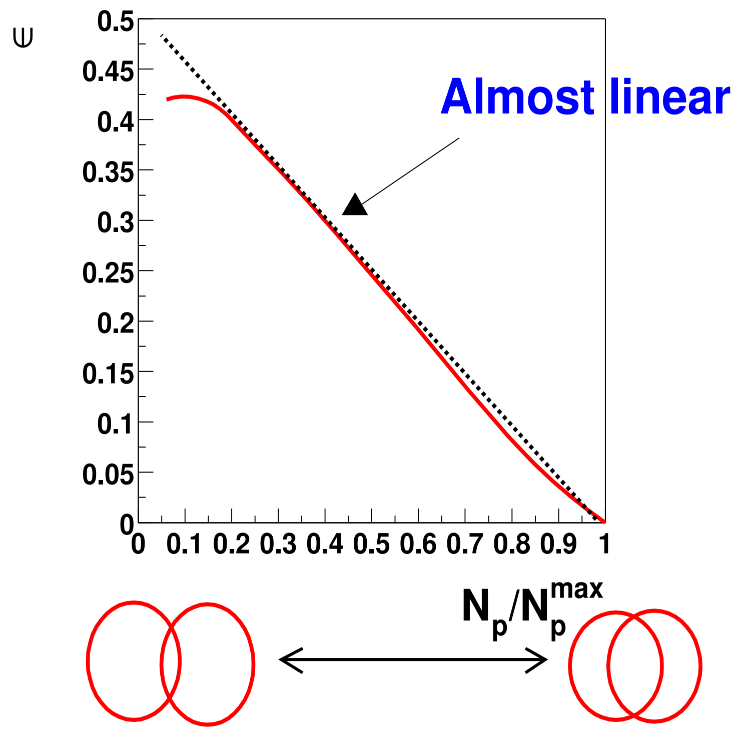
- $v_2(p_T)$ increases as a function p_T until $p_T \approx 2.0$ GeV and then flattens at $v_2 \approx 0.15$
- v_2 is large even at $p_T \approx 4.0\, GeV$. There is a 1.8 to 1 asymmetry between x and y.

Elliptic Flow is HUGE!!

Categorize the collision geometry:



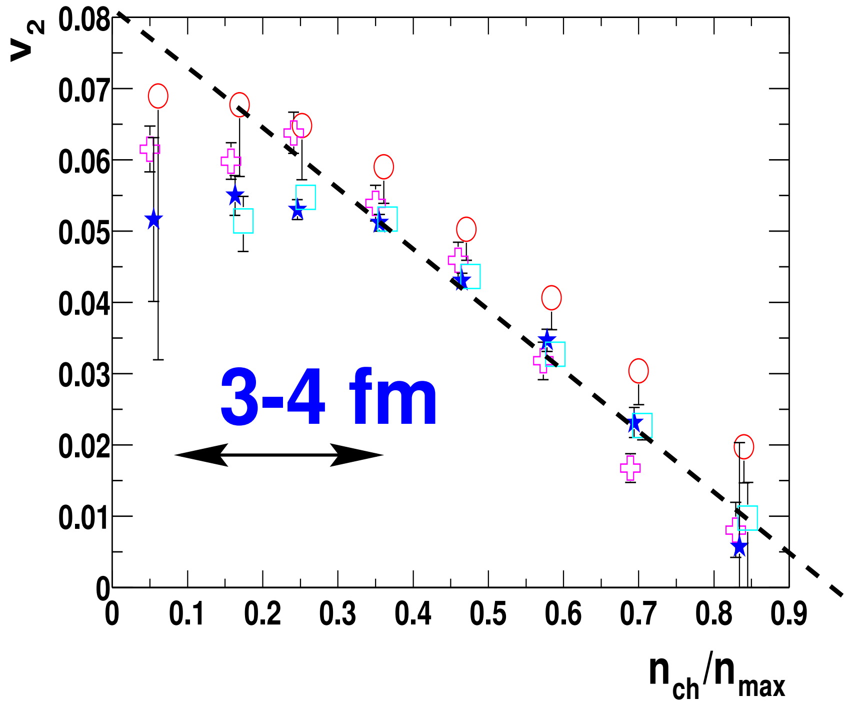
$$\in_x = \frac{< y^2 - x^2 >}{<y^2 + x^2>}$$

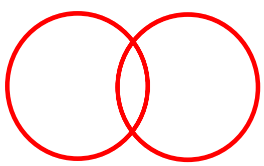


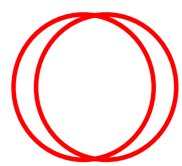
- $N_p \equiv$ is the number of nucleons that actually collide

Measurements of the integrated Elliptic Flow at RHIC:

Look at stars!







- If nothing changes as a function of centrality then expect:

  $v_2 \propto \epsilon$

- Up to corrections in periperal collisions: $v_2 \propto \epsilon$

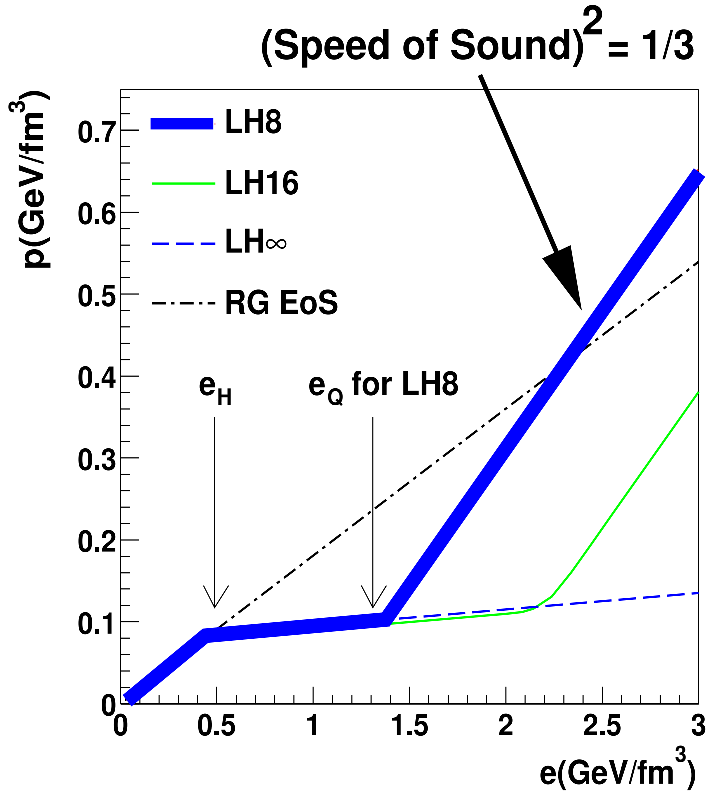
  These corrections "set in" on a scale of $\approx 3 - 4$ fm

Ideal Hydrodynamic Simulations of Heavy Ion Collisions:

- Assume the *Momentum Degradation Length* $\ell_{mfp} = 0$
- *Write the stress energy tensor:*

$$T^{\mu\nu} = (e+p)\,u^{\mu}u^{\nu} + p\,g^{\mu\nu}$$

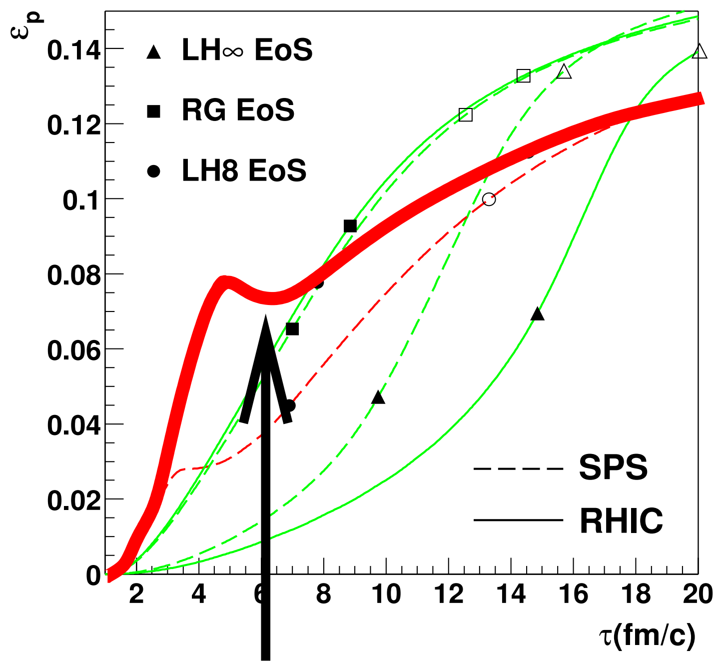
- *Input the equation of state:* $p(e)$
- *Solve the equations of motion:* $\partial_{\mu}T^{\mu\nu} = 0$



The hydrodynamic solution:

$$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle} \approx \text{“}v_2 \text{ weighted by } p_T^2$$

as a function time”

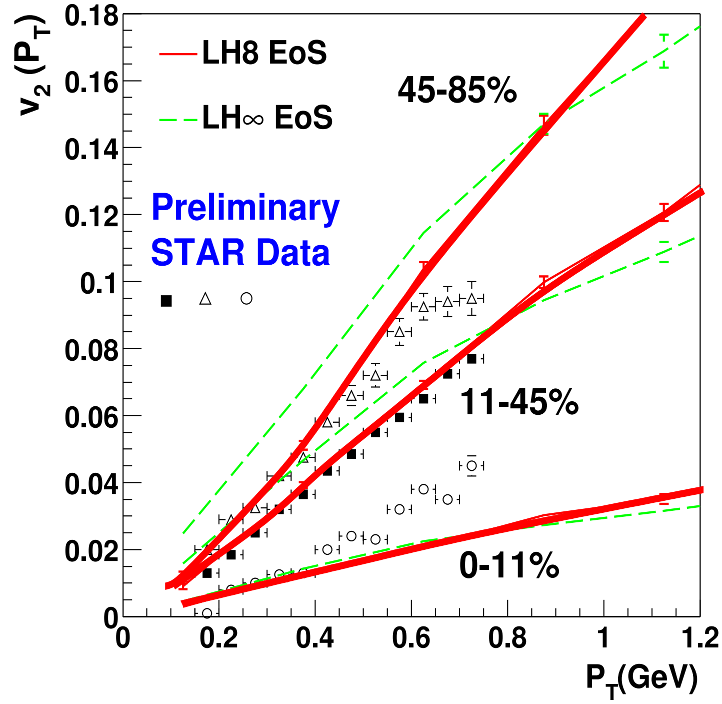


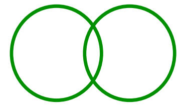
Goes into transition region at 5 fm/c

- v_2 rapidly rises during the plasma stage ≈ 5 fm/c
- v_2 then stalls in the transition region.
- Elliptic flow captures the early evolution.
- Much of the details of the subsequent evolution do not matter for v_2

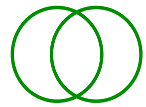
Calculate the thermal spectrum just below T_c

Compute $v_2(p_T) \equiv \langle \cos(2\,\phi) \rangle_{p_T}$

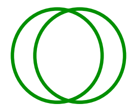




Hydro
Doesn't Work!

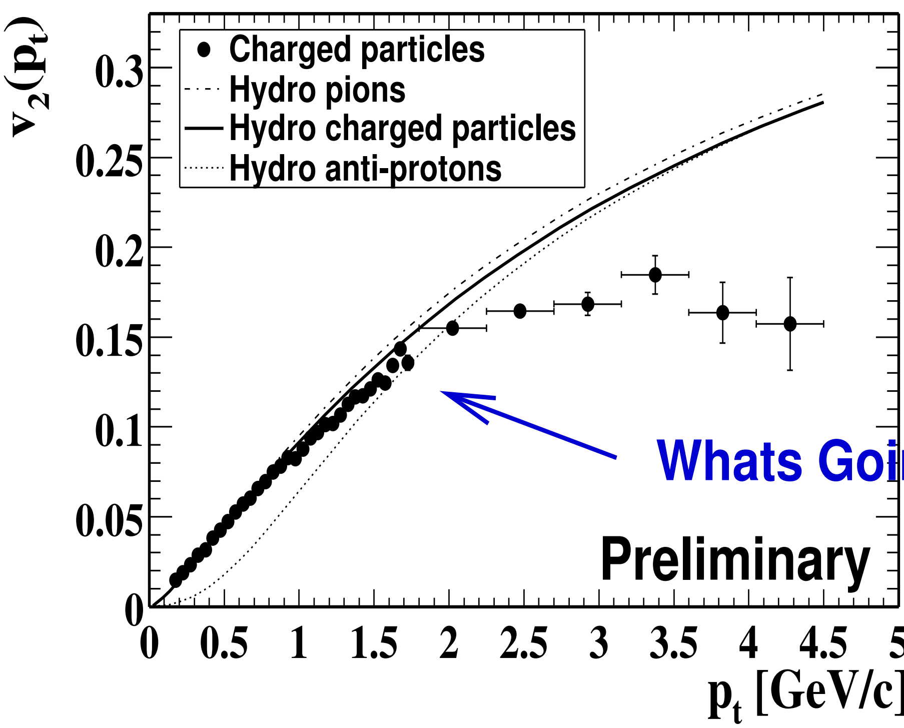


Hydro Works!



The data
has moved down
Hydro Works!

by Pasi Houvinnen



Whats Going on here?

Is Hydro believable?

- Hydro nicely explains the rise with p_T of elliptic flow.
- It fairly well reproduces the observed centrality dependence of elliptic flow.
- It fairs less well in peripheral collisions, at forward rapidity, and at lower energies where the multiplicity in the collision is smaller.

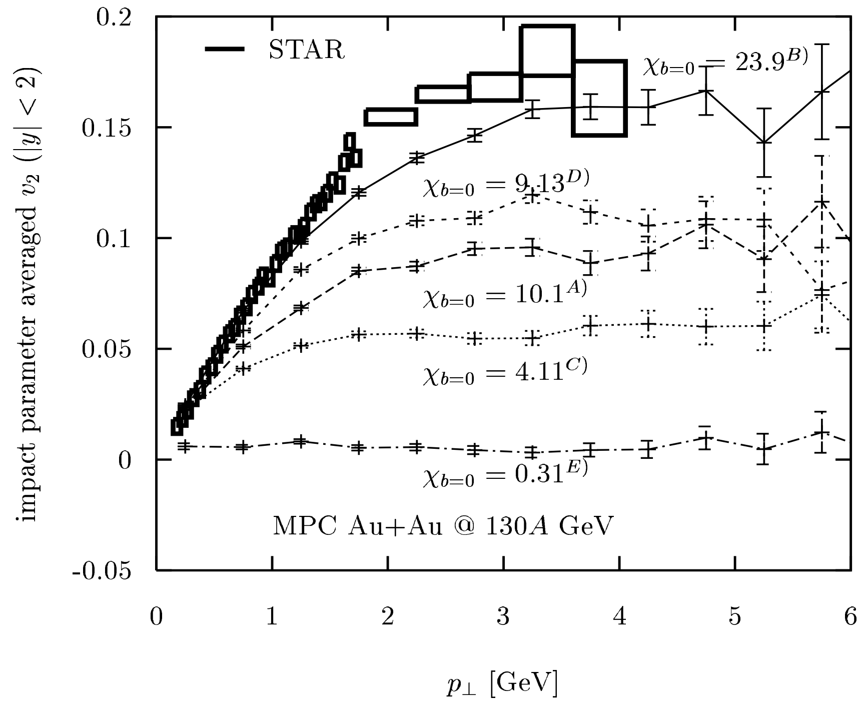
What changes if $\ell_{mfp} \neq 0$?

- If we need
$$\ell_{mfp} \ll \frac{1}{2\,\pi T}$$
in order to explain the observed elliptic flow, the hydrodynamic interpretation of elliptic flow must be abandoned!

Solution of the Boltzmann Equation (BE): Denes Molnar

1000 Classical Massless Particles with Constant X-Sections

$\sigma_0 \approx 10 - 20\,\text{mb}$



- The BE predicted a flattening of v_2 at high p_T.
- The observed v_2 breaks down consistently with viscous effects.
- 1. Can we understand this curve analytically ?
- 2. Why do the cross sections need to be

HUGE!??

How valid is Hydro? How much Entropy is produced?

$$\frac{d(\tau s)}{d\tau} = 0 \qquad \text{(Ideal Case)}$$
$$\frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T} \qquad \text{(Viscous Case)}$$

For hydrodynamics to be valid, the entropy produced over the time scale of the system, τ , must be small compared to the total :

$$\tau \, \frac{\frac{4}{3}\eta}{\tau T} \, \frac{1}{\tau s} \equiv \Gamma_s \frac{1}{\tau} \ll 1$$

- $\Gamma_s \equiv \frac{\frac{4}{3}\eta}{e+p}$ is the *Sound Attenuation Length*.
- Γ_s is the scientific way to talk about the mean free path.
- The mean free path should be less than the expansion rate $\frac{1}{\tau}$.

Estimates of η for the QGP and Heavy Ion Collisions

Perturbative QCD – Arnold, Moore, Yaffe.

- $\eta \approx 150\ T^3 \frac{1}{g^4}$.

  Based upon kinetic theory of quarks and gluons.

  Set $\alpha_s \to 1/2$ and $m_D \to$ a reasonable value

$$\left(\frac{\Gamma_s}{\tau}\right) \approx 0.4 \frac{1}{\tau T}$$

Strongly Coupled conformal N=4 SYM – Son, Starinets, Policastro

- No kinetic theory exists. Like most real liquids.

$$\left(\frac{\Gamma_s}{\tau}\right) = \frac{1}{3\pi}\frac{1}{\tau T} \approx 0.11 \frac{1}{\tau T}$$

Phenomenology – Molnar

Found could fit elliptic flow $v_2(p_T)$ only when

- $\frac{dN}{d\eta} = 1000$, $\sigma_0 = 10 \div 20$, and $\tau_o = 0.1$ fm.

$$\Gamma_s = 0.421 \frac{1}{n\sigma_0} \qquad \left(\frac{\Gamma_s}{\tau}\right) = 0.02 \div 0.04$$

- Constant cross section. Independent of time!

Best Guess: (At time τ_0)

With

$$T_o \sim 300\,\text{MeV} \quad \text{and} \quad \tau_0 \sim 1\,\text{fm}$$

Find:

$$\left(\frac{\Gamma_s}{\tau}\right) \approx 0.1 - 0.4$$

How does $\frac{\Gamma_s}{\tau}$ evolve?

Thermalization: How does Γ_s/τ evolve?

1. Bjorken Expansion | Scale Invariant Cross Section: $\sigma \sim \frac{\alpha_s^2}{T^2}$

- When entropy is conserved: $T \sim \frac{1}{\tau^{1/3}}$

$$\frac{\Gamma_s}{\tau} \sim \frac{\#}{\tau T} \sim \# \frac{1}{\tau^{2/3}}$$

$\Longrightarrow$ rapid thermalization

2. Bjorken Expansion | Constant Cross Section: $\sigma = \sigma_0$

- When particle number is conserved: $\tau n \sim$ Const

$$\frac{\Gamma_s}{\tau} \sim \frac{\ell_{m.f.p.}}{\tau} \sim \frac{1}{\tau n \sigma_0} \sim \text{Const}$$

$\Longrightarrow$ Constant thermalization

Spherical Expansion | Scale Invariant Cross Section: $\sigma \sim \frac{\alpha_s^2}{T^2}$

- Entropy conservation: $(sV) \sim$ Const and $s \sim T^3$.

  Then $T \sim \frac{1}{\tau}$.

$$\frac{\Gamma_s}{\tau} \sim \frac{\#}{\tau T} \sim \text{Const}$$

$\Longrightarrow$ Constant thermalization

Spherical Expansion | Constant Cross Section: σ_0

- Number Conservation: $(nV) \sim$ Const. $n \sim \frac{1}{\tau^3}$

$$\frac{\Gamma_s}{\tau} \sim \frac{\ell_{m.f.p.}}{\tau} \sim \frac{1}{\tau\, n\sigma_0} \sim \frac{\tau^2}{\sigma_0}$$

$\Longrightarrow$ rapid breakup.

Summary:

	σ	1 D Expansion	3 D Expansion
$\eta \propto T^3$	$\frac{\alpha_s}{T^2}$	++ $\sim \frac{1}{\tau^{2/3}}$	Const.
$\eta \propto T$	σ_0	Const.	-- $\sim \frac{\tau^2}{\sigma_0}$

Three models of viscosity:

- A scale free model: $\eta \propto T^3$

$$\eta = \frac{1}{5}\, s$$

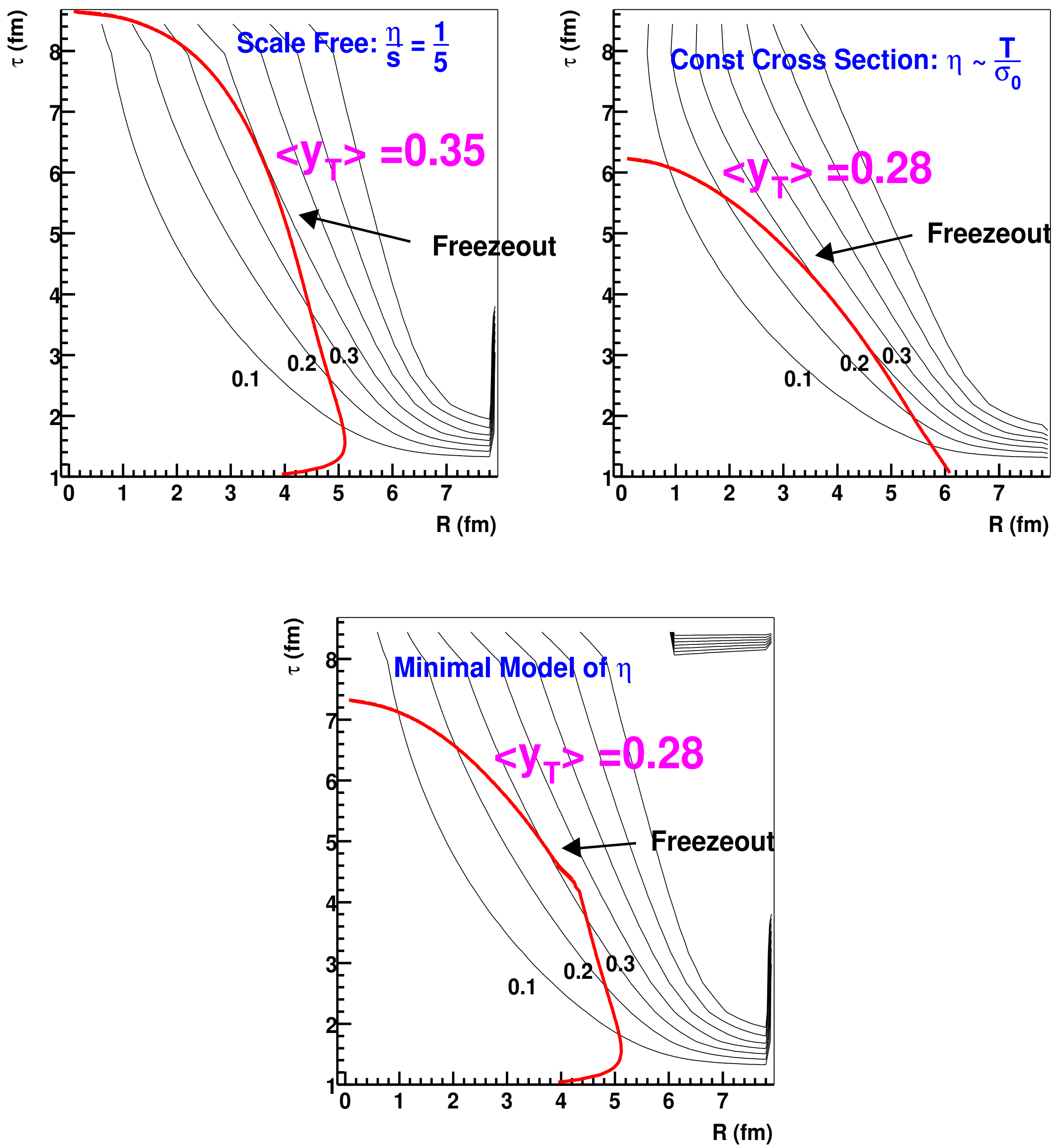
- Constant cross section:

$$\eta = 1.2\, \frac{T}{\sigma_0} \text{ with } \sigma_0 = 10 \text{ mb.}$$

- A minimal model: $e_c = 1$ GeV/fm^3

$$\eta = \begin{cases} 1.2\, \frac{\mathrm{T}}{\sigma_0} & \text{for } e < e_c \\ \frac{1}{5}\, \mathrm{s} & \text{for } e > e_c \end{cases}$$

Compare the three models of viscosity:



The minimal model of η and the Const X.-section model of η yield the same amount of radial flow.

Conclusions:

- Viscosity does not change the ideal hydrodynamic solution particularly much.
- Having a viscosity which is proportional to $\eta \sim T^3$ with a physically reasonable but small value of η/s reproduces the radial flow found by Denes with large cross sections and unphysical values of η/s.
- TO DO: Elliptic Flow
- TO DO: Compare viscous solutions with kinetic theory.

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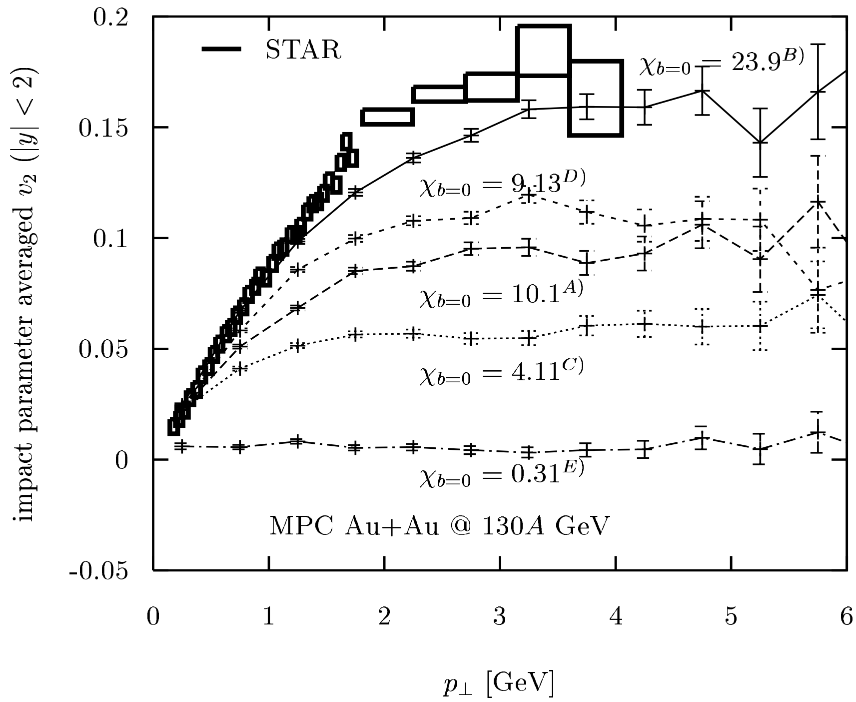

- The BE predicted a flattening of v_2 at high p_T.
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HUGE!??

How does viscosity change the thermal spectrum?

Perfect Thermal Equilibrium ℓ_{mfp}/L = 0

$$f_o = \frac{1}{\exp\left(\frac{p\cdot u}{T}\right) - 1}$$

Non-Equilibirium Effects $\ell_{mfp}/L \ll 1$ modify this distribution

- Finite size of the system
- Finite cross sections
- Expansion Dynamics – Gradients in Velocity

$$f \to f_o + \delta f$$

Calculate the non-equilibrium corrections δf.

These corrections modify Spectra and $v_2(p_T)$.

Want to calculate δf: Use the linearized Boltzmann equation

$$\frac{p^\mu}{E}\partial_\mu f_p = \int_{1,2,3} d\Gamma_{12\to 3p}\,(f_1 f_2 - f_3 f_p)$$

Linearize the Boltzmann equation:

- Substitute $f \to f^e + \delta f$ with $f_p^e = e^{-pu/T}$
- Keep first order in gradients.
- Use equilibrium: $f_1^e f_2^e = f_3^e f_4^e$

$$\frac{p^\mu}{E}\partial_\mu f_p^e = \int_{1,2,3} d\Gamma_{12\to 3p}\, f_1^e f_2^e \left[\frac{\delta f_1}{f_1^e} + \frac{\delta f_2}{f_2^e} - \frac{\delta f_3}{f_3^e} - \frac{\delta f_4}{f_4^e}\right]$$

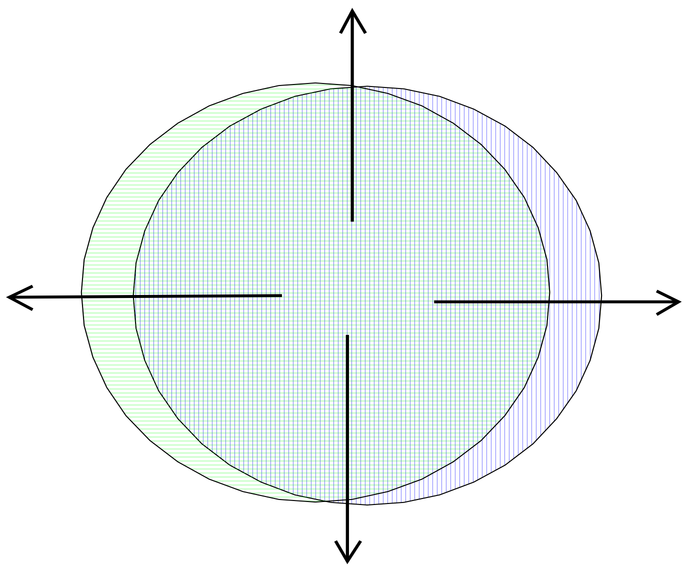
This is an integral equation for δf.

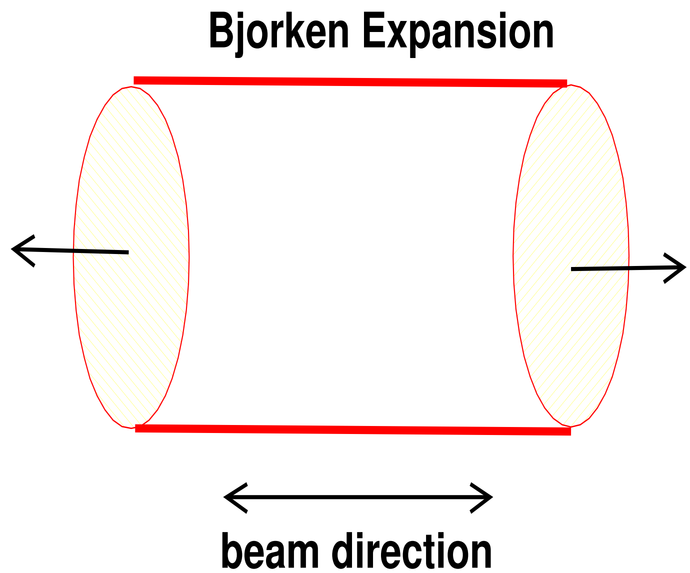
- Approximate δf with the first Chapman Enskog approximation:

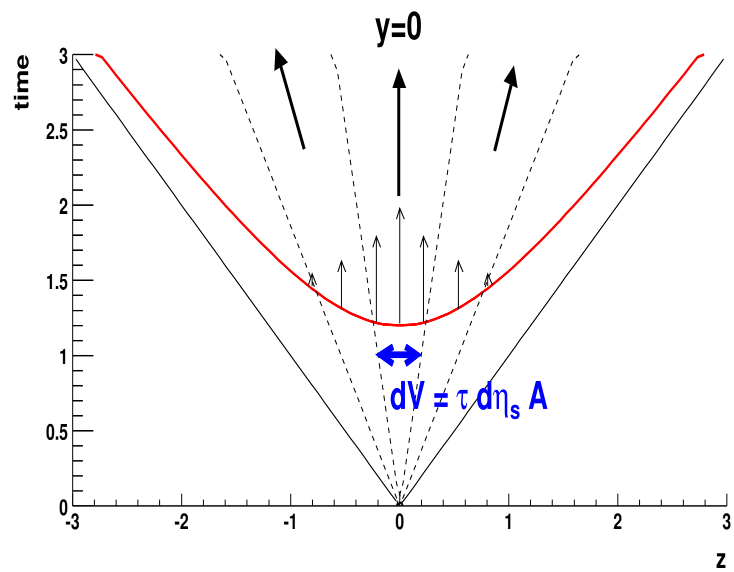
$$\delta f = f_o\left(\frac{p\cdot u}{T}\right)\ \underbrace{C}_{\Gamma_s}\ \frac{p_\mu p_\nu}{T^2}\ \underbrace{\langle\partial^\mu u^\nu\rangle}_{\frac{1}{\tau}}$$

Viscous corrections grow as: $p_T^2 \times \frac{\Gamma_s}{\tau}$

Parametrize Ideal Hydrodynamic Solutions:

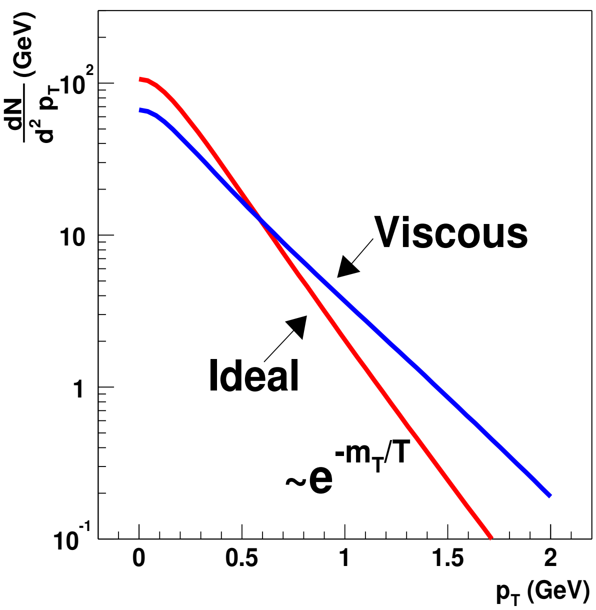






Now compute the thermal spectrum of particles at y=0:

Viscous corrections to the spectrum: Qualitative



The transverse pressure is larger with viscosity:

$$
\begin{aligned}
T_{zz} &= p - \frac{\mathbf{4}}{\mathbf{3}}\frac{\eta}{\tau} \\
T_{xx} = T_{yy} &= p + \frac{\mathbf{2}}{\mathbf{3}}\frac{\eta}{\tau}
\end{aligned}
$$

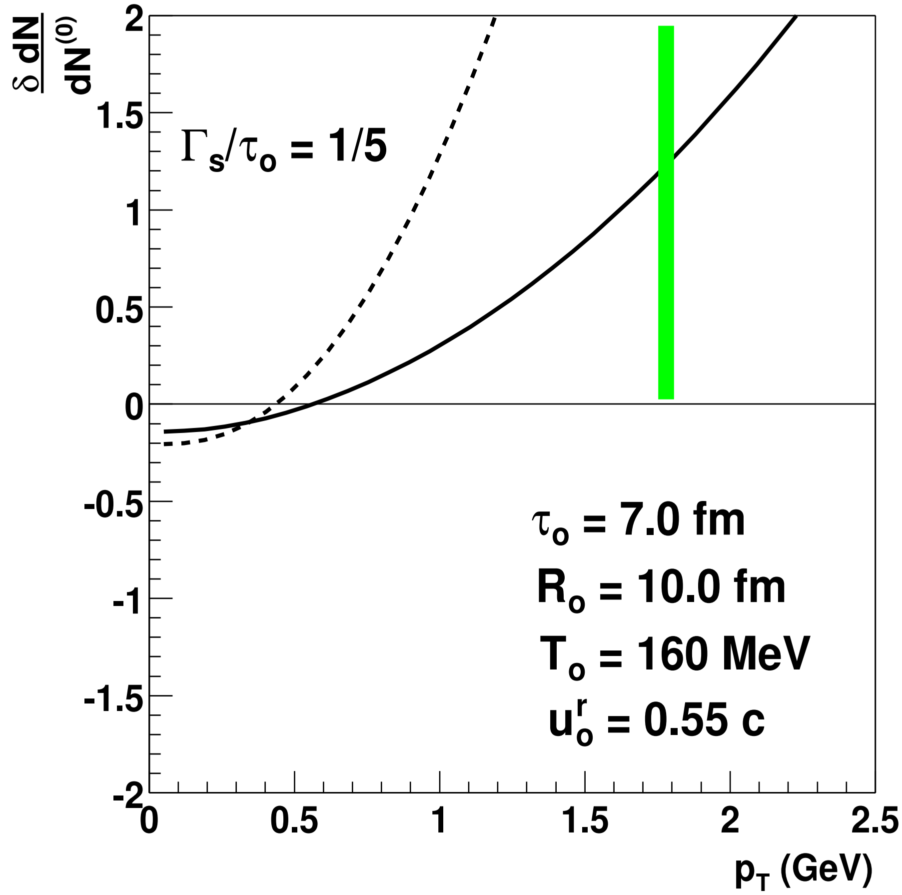
Viscous corrections to p_T spectrum: Quantitative

$$dN_o + \delta\, dN = \int p^\mu d\Sigma_\mu\, f_o + \delta f$$

Now you can do the calculation:

$$
\begin{aligned}
\frac{\delta\, dN}{dN_o} &= \frac{\Gamma_s}{4\tau}\left\{\left(\frac{p_T}{T}\right)^2 - \left(\frac{m_T}{T}\right)^2 \frac{1}{2}\left(\frac{K_3(\frac{m_T}{T})}{K_1(\frac{m_T}{T})} - 1\right)\right\} \\
&\to \frac{\Gamma_s}{4\tau}\left(\frac{p_T}{T}\right)^2
\end{aligned}
$$

When the correction becomes O(1) we are supposed to stop.

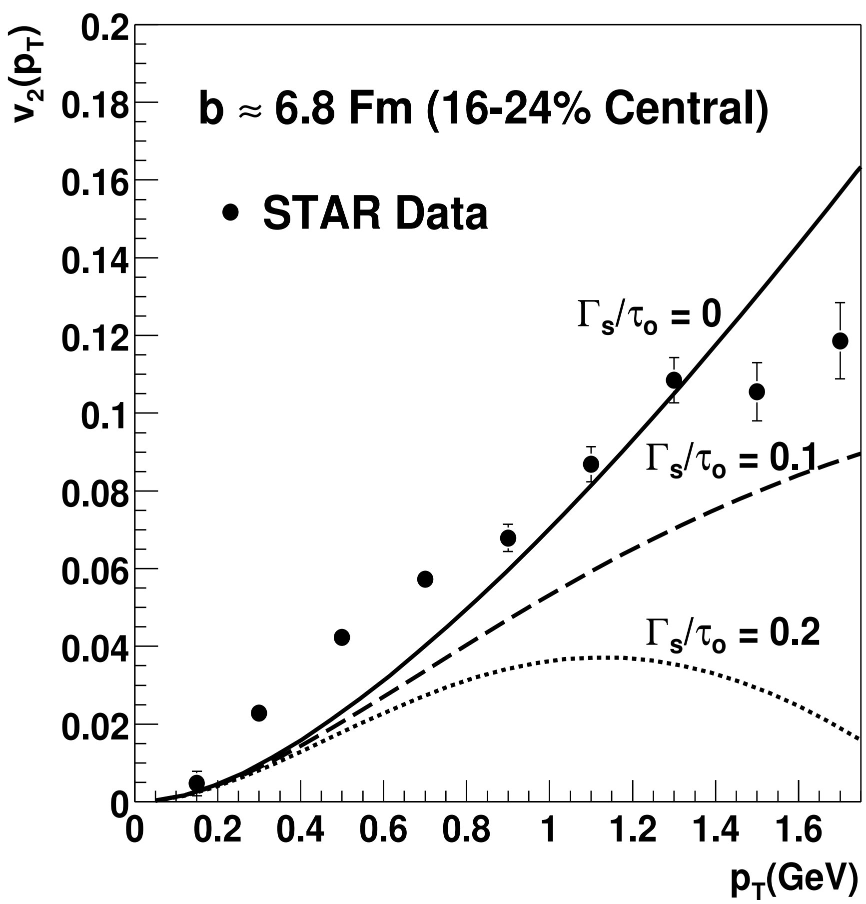


$$\frac{\delta dN}{dN^{(0)}} \equiv \frac{\frac{dN^{(1)}}{p_T\, dp_T\, dy}}{\frac{dN^{(0)}}{p_T\, dp_T\, dy}}$$

The maximum possible p_T accessible to Hydrodynamics is ~ 1.8 GeV – A couple of times T_{eff} .

Does viscosity set the scale for the turnover?

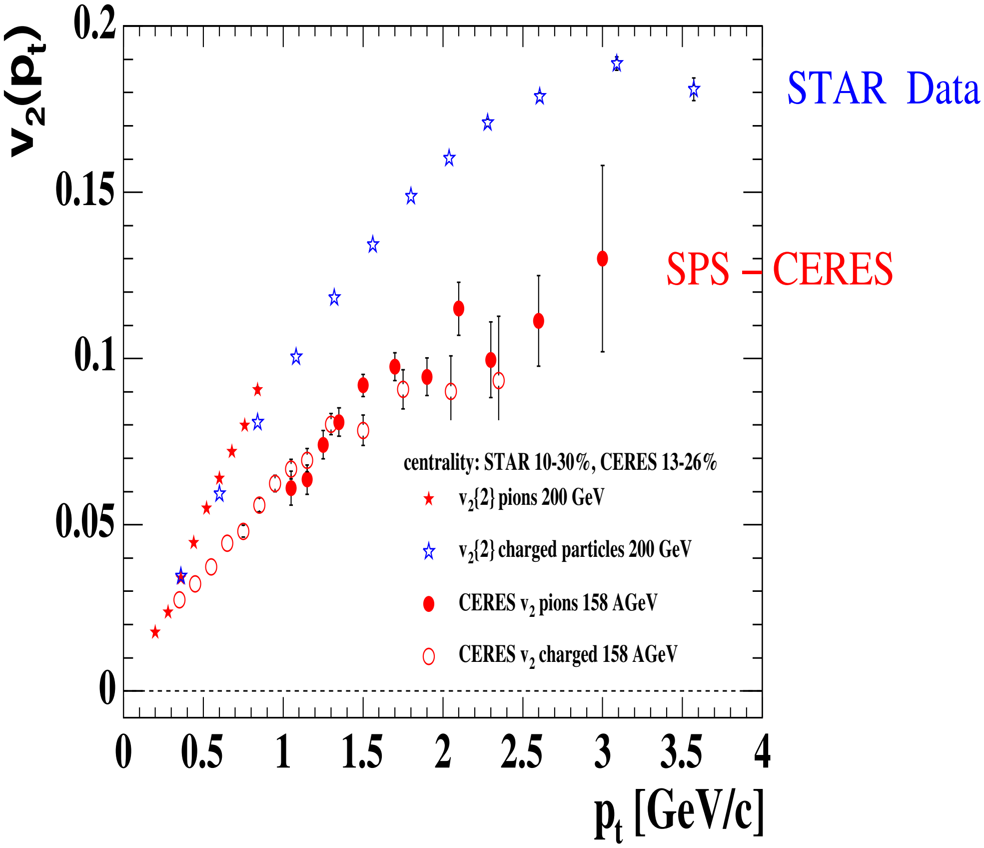
- Once δf is computed viscous corrections to all observables of the "Blast Wave Model" may be computed.



- The shape is not perfect. But viscosity does set the scale.

- More complete calculations are in the works.

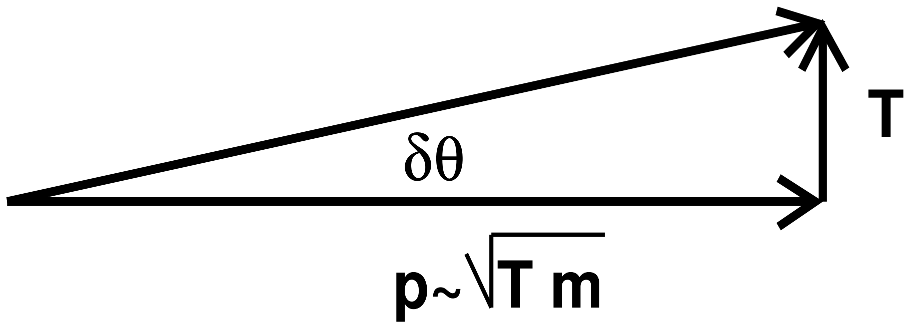
Compare the SPS and RHIC



- v_2 flattens earlier at the SPS?
- What about forward rapidities at RHIC?

Will the charm quark thermalize?

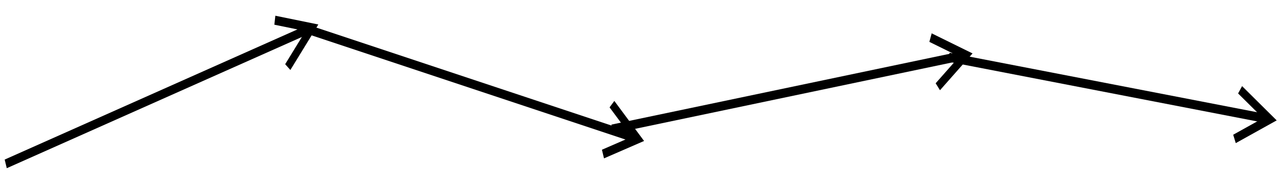
- In collaboration with Guy Moore



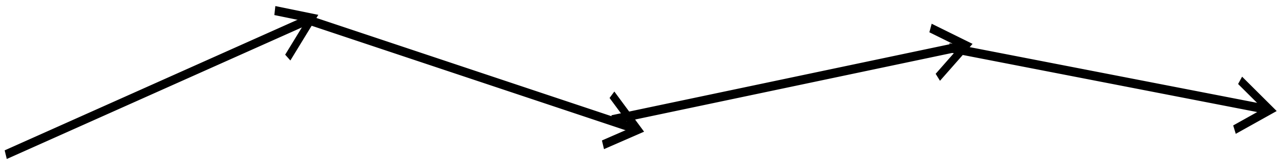
$$\delta\theta \sim \sqrt{\frac{T}{m}}$$

- The collision only scarcely changes the direction of the charm quark

- The charm quark undergoes a random walk suffering <u>many</u> collisions provided $\ell_{m.f.p} \ll L$



$$(\Delta\theta)^2 \sim N_{kick}\,(\delta\theta)^2 \sim N_{kick}\,\frac{T}{m}$$



$$(\Delta\theta)^2 \sim N_{kick}\,(\delta\theta)^2 \sim N_{kick}\,\frac{T}{m}$$

- For equilibration we need:

$$(\Delta\Theta)^2 \sim 1 \qquad \text{or} \qquad N_{\text{kick}} \sim \frac{M}{T}$$

- Thus for charm equilibration we have:

$$\begin{aligned} \tau_R^{\text{charm}} &\sim \frac{M}{T}\tau_R^{\text{light}} \\ &\sim \frac{M}{T}\frac{\eta}{e+p} \end{aligned}$$

It takes a longer time to equilibrate charm.

If you think you know η you should be able to compute the charm spectrum.

Langevin description of heavy quark thermalization

- When the number of kicks to the heavy quark is large we can replace the interaction by random kicks: $\xi(t)$.

$$\langle \xi(t)\xi(t') \rangle = \kappa\delta(t-t') \, .$$

- κ is the mean squared momentum transfer per time.

- Add a damping term $-\eta_D$p.

$$\frac{dp}{dt} = \xi(t) - \eta_D p$$

η_D^{-1} is the equilibration time τ_R^{charm}

Relating the random noise to the damping: FDT

$$\frac{dp}{dt} = \xi(t) - \eta_D p$$

The solution to this equation is simple:

$$p(t) = p_0 e^{-\eta_D t} + \int_{-\infty}^{t} dt' e^{\eta_D (t'-t)} \xi(t') \,,$$

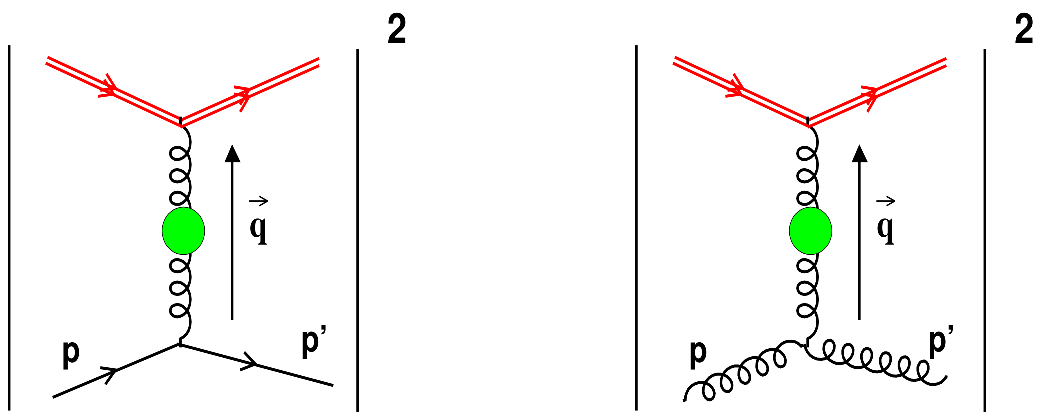
Now in equilibrium statistical mechanics:

$$3MT \quad = \quad \lim_{t\to\infty} \left\langle p^2(t) \right\rangle = \frac{\kappa}{2\eta_D}$$

Once I compute κ, I know the relaxation time η_D^{-1}

Computing κ in the perturbative QGP

- κ is the average momentum squared transferred to the particle per unit time:



$$\kappa = \int_{\mathbf{p},\mathbf{p}',\mathbf{q}} \mathbf{q}^2 \left[f(p)(1+f(p')) \left| M_{\text{glue}} \right|^2 \right]$$

The Relaxation Time Is:

$$\eta_D^{-1} = \text{A Number} \times \frac{M}{T} \times \frac{\eta}{e+p}$$
$$= \underbrace{6}_{\text{BIG!}} \times 6 \times (0.1 \div 0.4 \text{ fm})$$
$$\approx 3 \div 12 \text{ fm}$$

Why is the factor big?

- The shear viscosity relaxes the tensor $T^{\mu\nu}$

  – angular momentum $\ell = 2$.

- Diffusion relaxes a vector J^{μ}

  – angular momentum $\ell = 1$

- Roughly:

  $\tau_R \sim \frac{1}{\ell(\ell+1)}$ – Gives a factor of ≈ 3

- Diffusion relates to Quarks while shear relates to gluons. Find an additional factor of charges: $C_A/C_F \approx 2.5$

Equilibration in an expanding medium

- The relevant parameter is:

$$\chi = \int_{\tau_0}^{\tau} d\tau' \eta_D(\tau')$$

- For a Bjorken expansion:

$$T \propto \left(\frac{\tau}{\tau_0}\right)^{-1/3} \quad \text{and} \quad \eta_D \propto T^2 \propto \left(\frac{\tau}{\tau_0}\right)^{-2/3}$$

χ increases slowly with time: $\chi \propto \left(\frac{\tau_0}{\tau}\right)^{1/3}$

- Substituting – $\tau_0 \approx 0.5$ fm, $\tau_f \approx 6$ fm, $T_0 \approx 300$ MeV

$$\chi \approx 0.2 \div 0.8 \quad \text{for} \quad \frac{\eta}{s} \approx 0.1 \div 0.4$$

The charm quark may thermalize slightly

How to calculating the change in the spectrum?

- Rewrite the Langevin equation as a Boltzman-Fokker Plank Equation.

$$\underbrace{\frac{\partial P}{\partial t} + \frac{p^i}{m}\frac{\partial P}{\partial x^i}}_{\text{Boltzmann Drift Term}} \quad = \underbrace{\left[\frac{\partial}{\partial p_i}\eta_D p_i + \frac{\partial^2}{\partial p^2} MT\,\eta_D\right] P(p,\tau)}_{\text{Momentum Drift and Diffusion Term}}$$

- Can then find the greens function of the Fokker Plank Equation for a Bjorken Expansion:

$$P(p|p_0,\tau,\tau_0)\ .$$

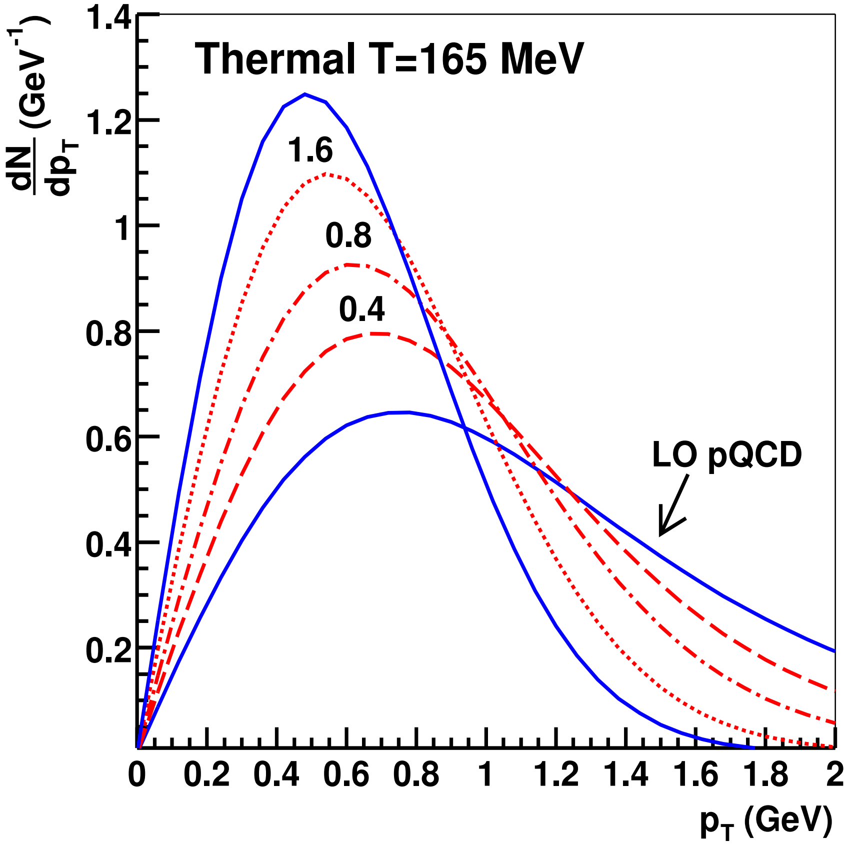
- $P(p|p_0,\tau,\tau_0)$ is the probability to find a heavy quark with momentum p at time τ given that it had momentum p_0 at time τ_0.

$$P(p|p_0,t,t_0) \sim \frac{1}{\sqrt{2\pi M T_\perp(\chi,\tau)}} \exp\left[-\frac{(p-p_0 e^{-\chi})^2}{2MT_\perp(\chi,\tau)}\right]$$

Now Convolve the Green's Function with the initial conditions

$$\frac{d^2N}{d^2p_\perp} \;=\; \int d^2p^0_\perp \; P(p_\perp|p^0_\perp,\tau,\tau_0)\;\frac{dN^{(0)}}{d^2p^0_\perp}$$

- The spectrum as a function of $\chi = \int_{\tau_0}^{\tau_f} d\tau' \eta_D(\tau')$



- $\chi \approx 0.0 \div 0.8$

If any modification of the low p_T spectrum is seen it suggests that the viscosity is quite small

Conclusions:

- In perturbation theory the relaxation of heavy quarks is a factor of 40 different from the relaxation of viscosity.
- The reasons seem "generic"
- Given an estimate of the $\frac{\eta}{e+p} \approx \frac{1}{5T}$ we expect only small modifications to the charm quark spectrum. Can they be seen?
- If the shear viscosity is $\frac{\eta}{e+p} = \frac{1}{4\pi T}$ then $\chi \approx 0.8$. Some modifications to the spectrum should be visible.
- Look at the slope of the spectrum between $\sqrt{MT}$ and M.
- For a given value of χ what is the elliptic flow?

If charm hadrons show the same v_2 as all others then hydrodynamics is not responsible for the observed elliptic flow.